**event-triggered Coordinated path following control for underactuated surface vehicles with input saturation**

# Abstract

# Introduction

# PRELIMINARIES and Problem Formulation

## Graph theory

## Vehicle’s model

Consider a group of underactuated surface vehicles labeled from 1 to N. The kinematic equation of th vehicle can be expressed as:

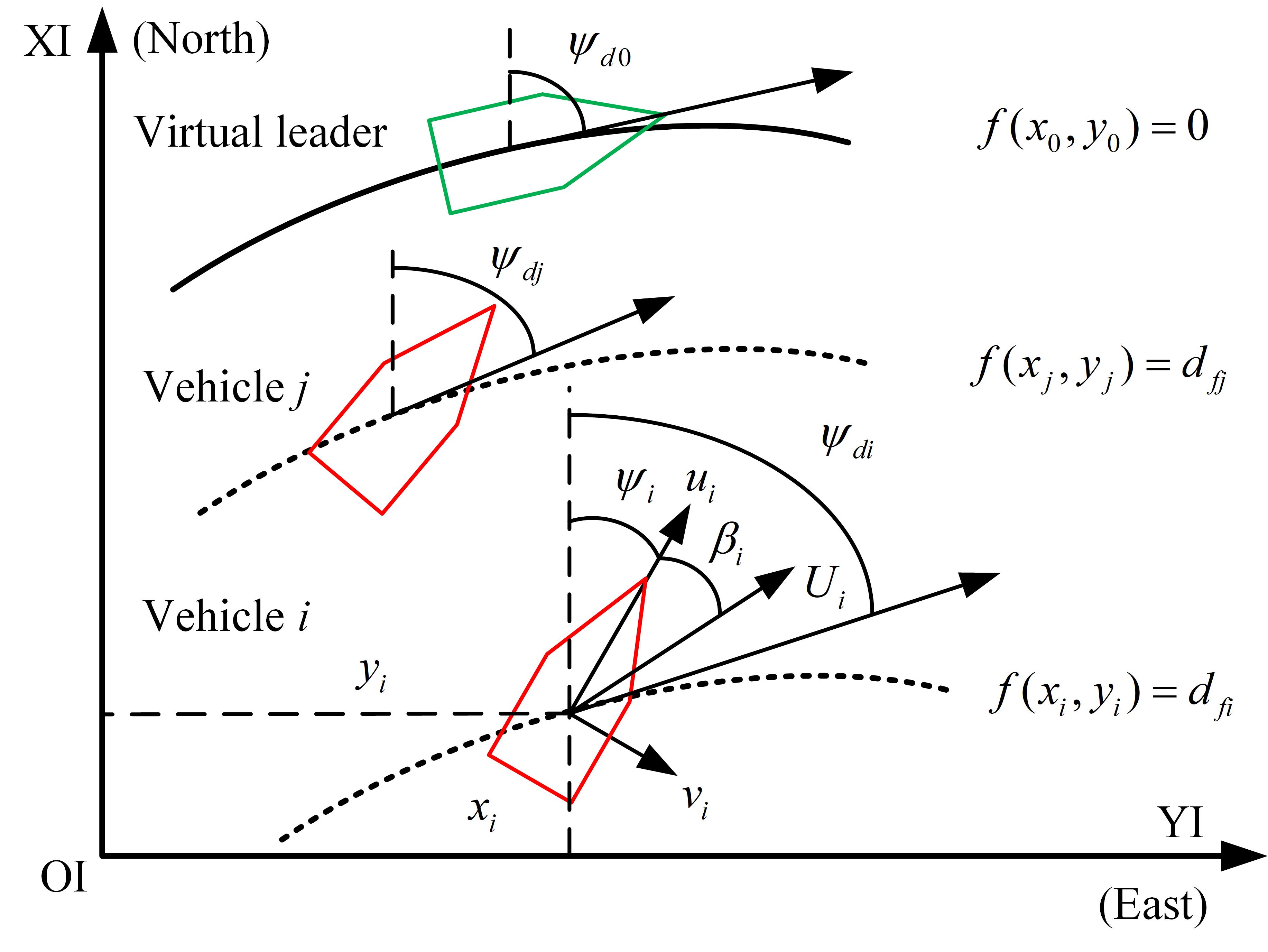
(1)

The kinetic equation of th vehicle can be expressed as:

(2)

where

(3)



*Fig. 1. The geometric illustration of coordinated path following*

## Problem formulation

The desired path can be represented by an implicit function as follows:

(4)

Then the path following error of the th vehicle can be described by . The desired orientation of th vehicle is calculated by:

(5)

The derivative of can be obtained as

(6)

The main objective is to force the errors and to satisfy predefined constraints, so that the vehicles will locate on the different level set of the desired path. Simultaneously, we regulated the vehicles’ dynamic behavior according to the difference of arc length between two different vehicles which can be calculated by:

(7)

The coordinated path following control objective can be concluded as follows:

O1) path-following task: Lead a fleet of surface vehicles to navigate along the different level sets of the predefined geometric path without time constraints which can be described as:

(8)

where is a constant.

O2) dynamic formation task: Drive all the vehicles to form a designated formation by regulating the course speed which can be described as

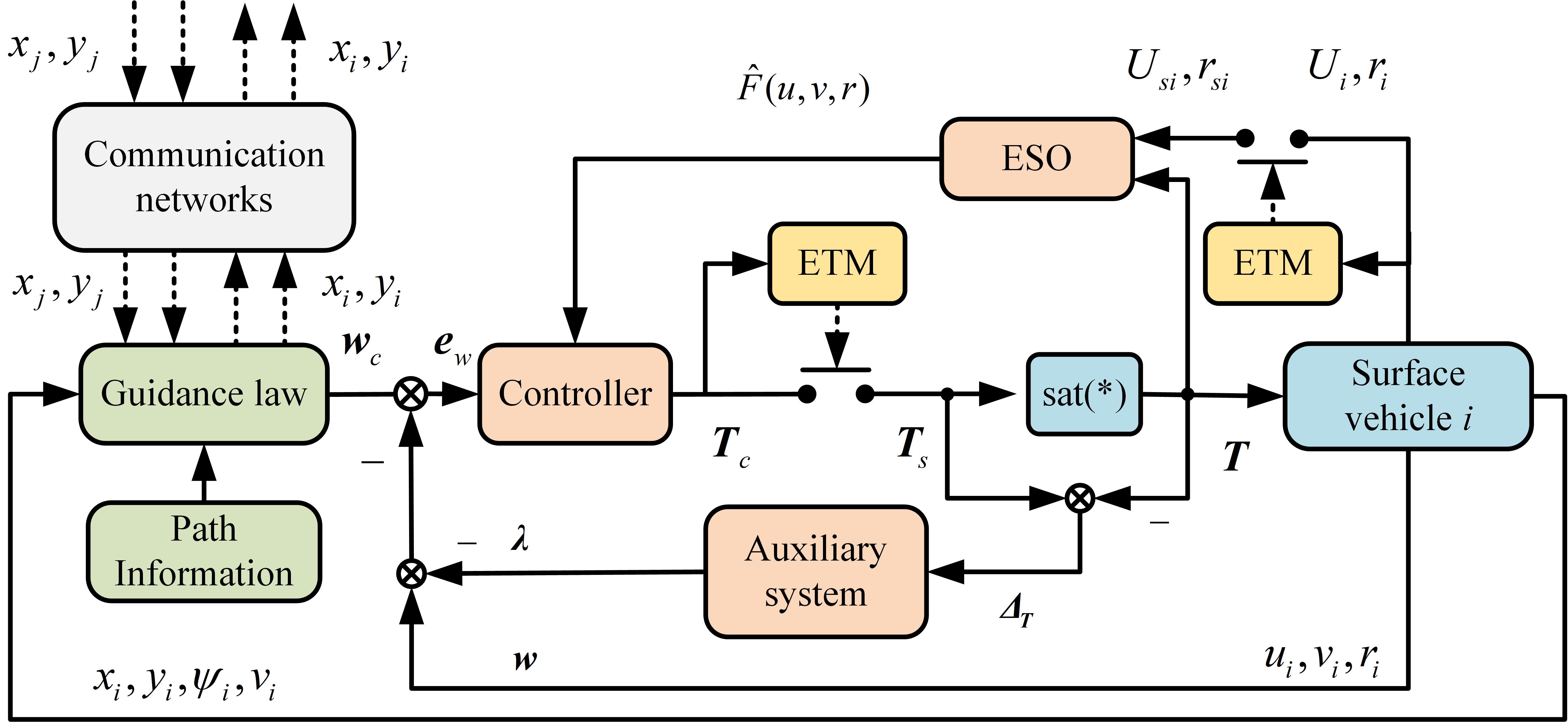
(9)

In addition, each vehicle’ course speed should satisfy the desired speed assignment

(10)

where and are positive constants.

# coordinated cotroller desigen



*Fig. 2. Block diagram of the coordinated path following control system*

## Coordinated guidance law design

According to the control objective (O1) and (O2), the design process of the coordinated guidance law can be divided into two steps. In the first step, the coordinated path following guidance law will be designed to lead each vehicle to follow the corresponding level set of the desired path without time constraints. In the second step, the dynamic formation guidance law is designed to force the vehicles to satisfy the expected formation with temporal constraints. Before the design process, a virtual leader on the desired path is introduced as

(11)

where is the heading angle of the virtual leader which can be calculated by (5). The virtual leader will always on the desired path if the initial values of and locate on the desired path. Then, the design procedure is elaborated in the following two steps.

**Step1.** Denote the coordinated path following error of th vehicle as :

(12)

Let , . The coordinated path following error can be rewrite as the following compact form

(13)

The derivative of is calculated as

(14)

Then, the desired heading angle of th vehicle can be designed as

(15)

where , is a positive constant. Define the tracking errors and as and , where and are the filtered version of and , and are the virtual control law and auxiliary state which will be designed in the next subsection. The filtered errors are defined as and . The first order filters are introduced as

(16)

The errors’ dynamics of filters are calculated as

(17)

Then, the derivative of can be obtained as

(18)

The desired yaw angular velocity is designed as

(19)

**Step2.** Define the dynamic formation error of th vehicle as

(20)

where is the arc length between th vehicle and th vehicle, is the arc length between th vehicle and virtual leader. can be calculated as

(21)

The derivative of is calculated as

(22)

where , . Let , . Then the error dynamics can be written as the following compact form.

(23)

Then, the desired course speed of th vehicle can be designed as

(24)

## Event triggered controller design

In this subsection, the event triggered controller will be designed to force the course velocity and the yaw angular velocity of th vehicle to track the desired guidance signals and designed in last subsection. Firstly, the ET-ESO is introduced to estimate the unknown disturbances. Then, based on the estimated information generated by ET-ESO, the proportional feedback controller is developed by backstepping method. Lastly, the event triggered mechanism is designed to reduce the operations of thrusters.

**Step1.** Define the estimated errors as , . According to (3), ESO can be designed as

(25)

where . The ETM for and is designed as:

(26)

(27)

**Step2.** Considering the input saturation, we can rewrite the kinetic equation as

(28)

where , is the measured value of . To compensate the input saturation, the dynamic auxiliary system is designed as

(29)

Let be the desired guidance signal, be the filtered version of .The first order differentiator is introduced to obtain the derivative of as

(30)

where , . Define the tracking error and filtered error as and . The error dynamics of and are calculated as

(31)

(32)

Then, the proportional feedback control law is designed as

(33)

The control law is completed here. To reduce the operations of thrusters, the ETM for controller will be designed in next step.

**Step3.** Let , . The ETM for can be designed as

(34)

Let .Then, the ETM for is designed as

(35)

## Stability analysis

**Step1.** We analyze the stability of kinematic tracking errors , and in this step. Let , , , , , . Substituting the virtual control law (15), (19) and (24) into (14), (18) and (23) respectively, we can get

(36)

where , , , , , .

Lemma 1. Considering the

Proof: Construct Lyapunov function as . Taking the derivative of along subsystem (36), we can obtain that

**Step2.** The stability of ESO is analyzed in this step. According to the property of the ETM described as (26) and (27), the can be expressed by

(37)

where , , . Combined (3), (25) and (37), the estimated errors’ dynamics can be calculated as

(38)

Let , . The error dynamics (38) can be rewritten as

(39)

Lemma 2.

**Step3.** We analyze the stability of kinetic tracking errors , and . According to the property of the ETM described as (26) and (27), the can be expressed by

Lemma 3.

Based on the above lemmas, we give the following stability theorem of the whole closed-loop control system.

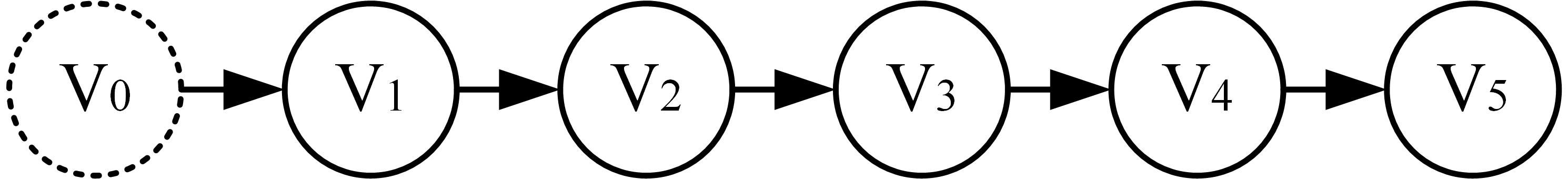
Theorem 1.

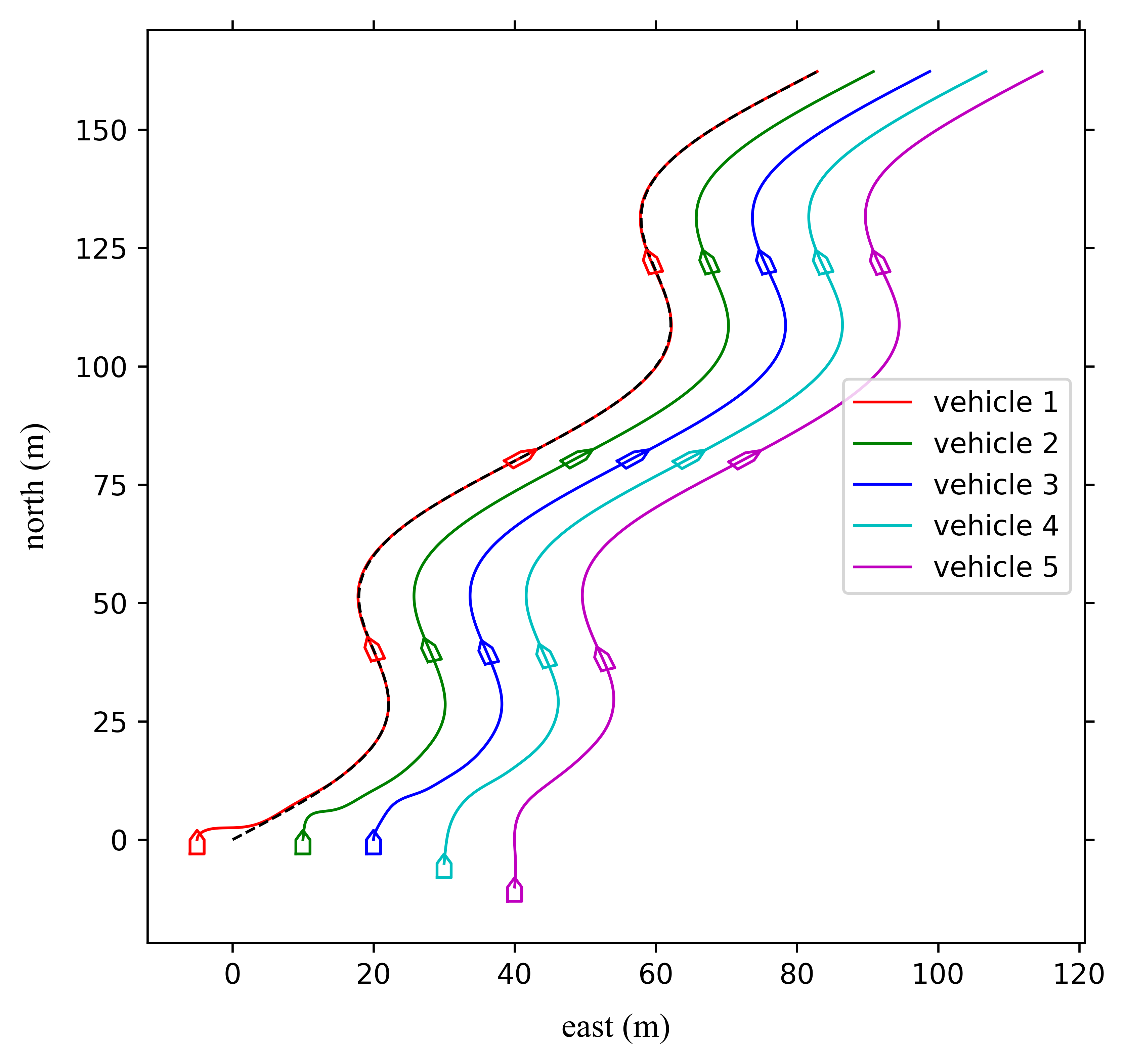
In addition, we will proof the Zeno phenomenon will be avoided by the ETM (26), (27) and (35).

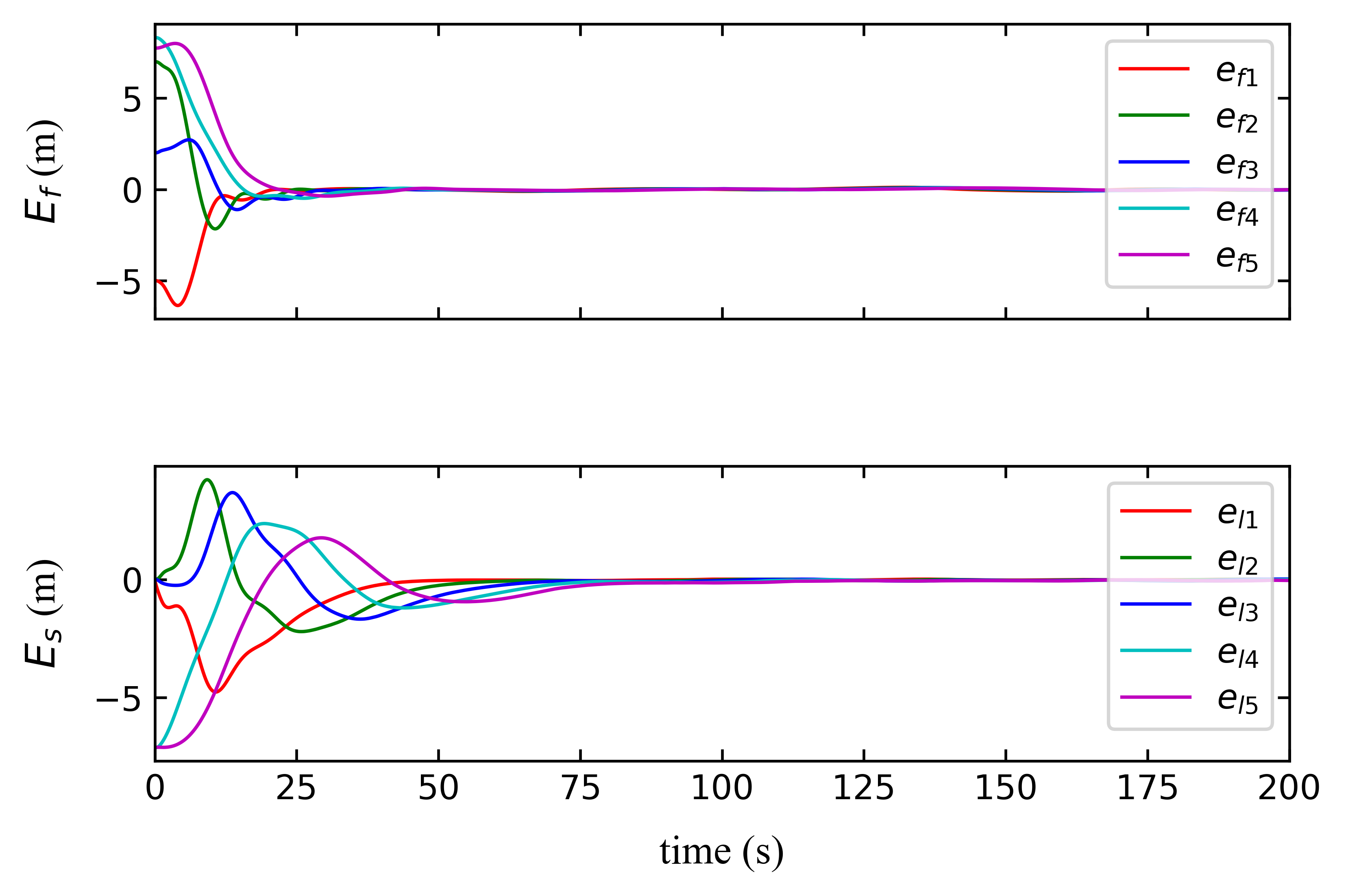
Theorem 2.

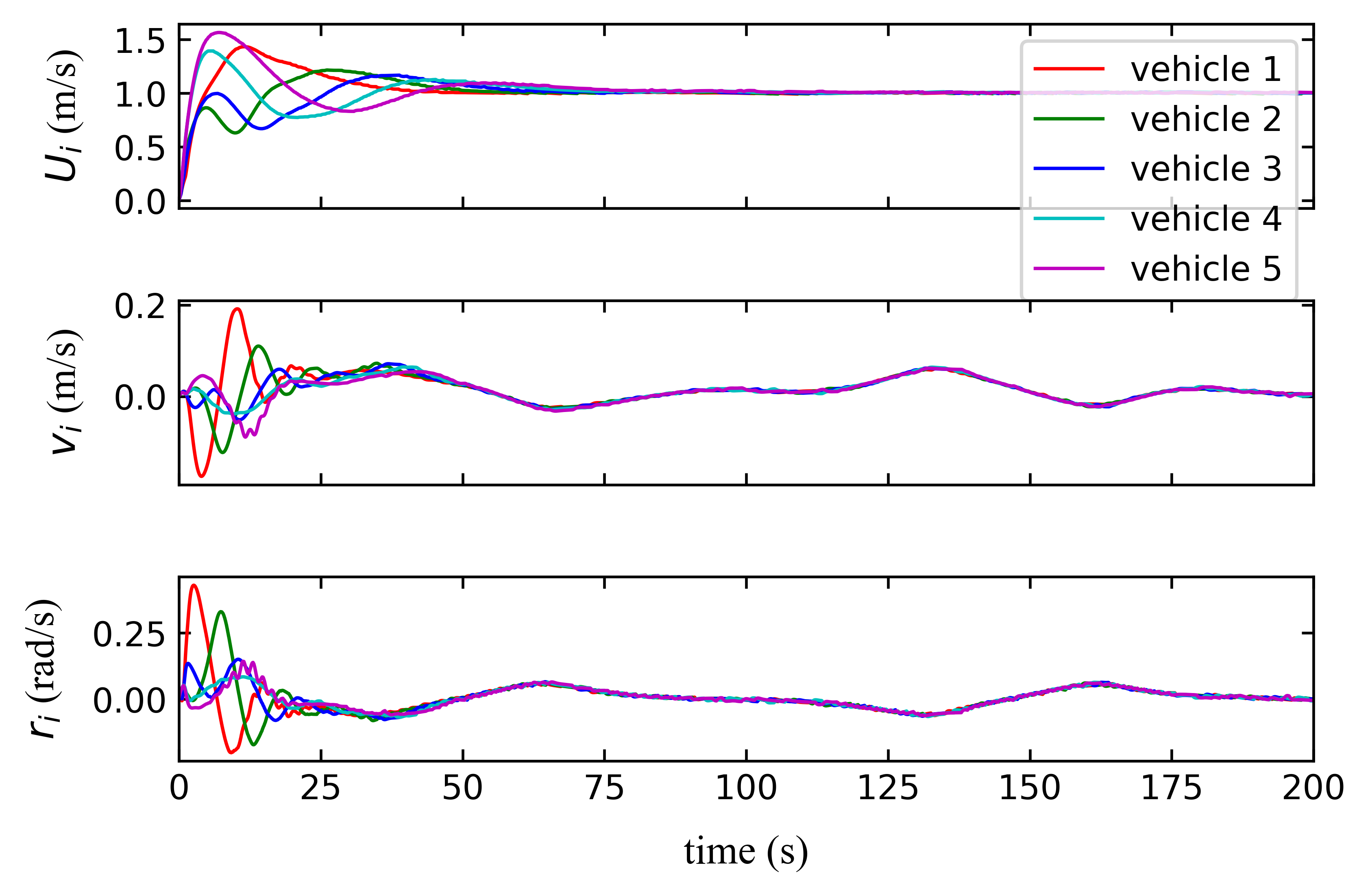
# Simulation

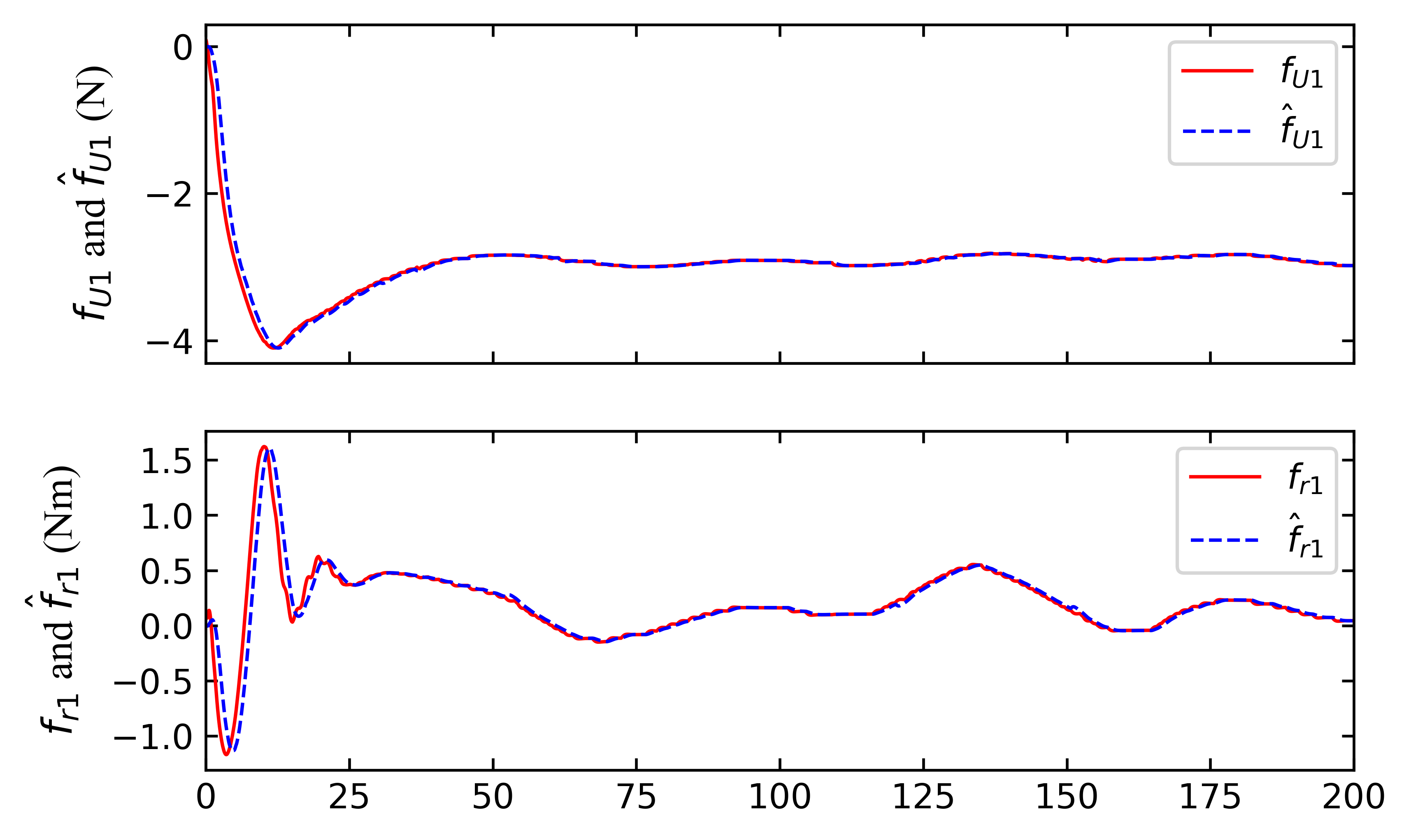
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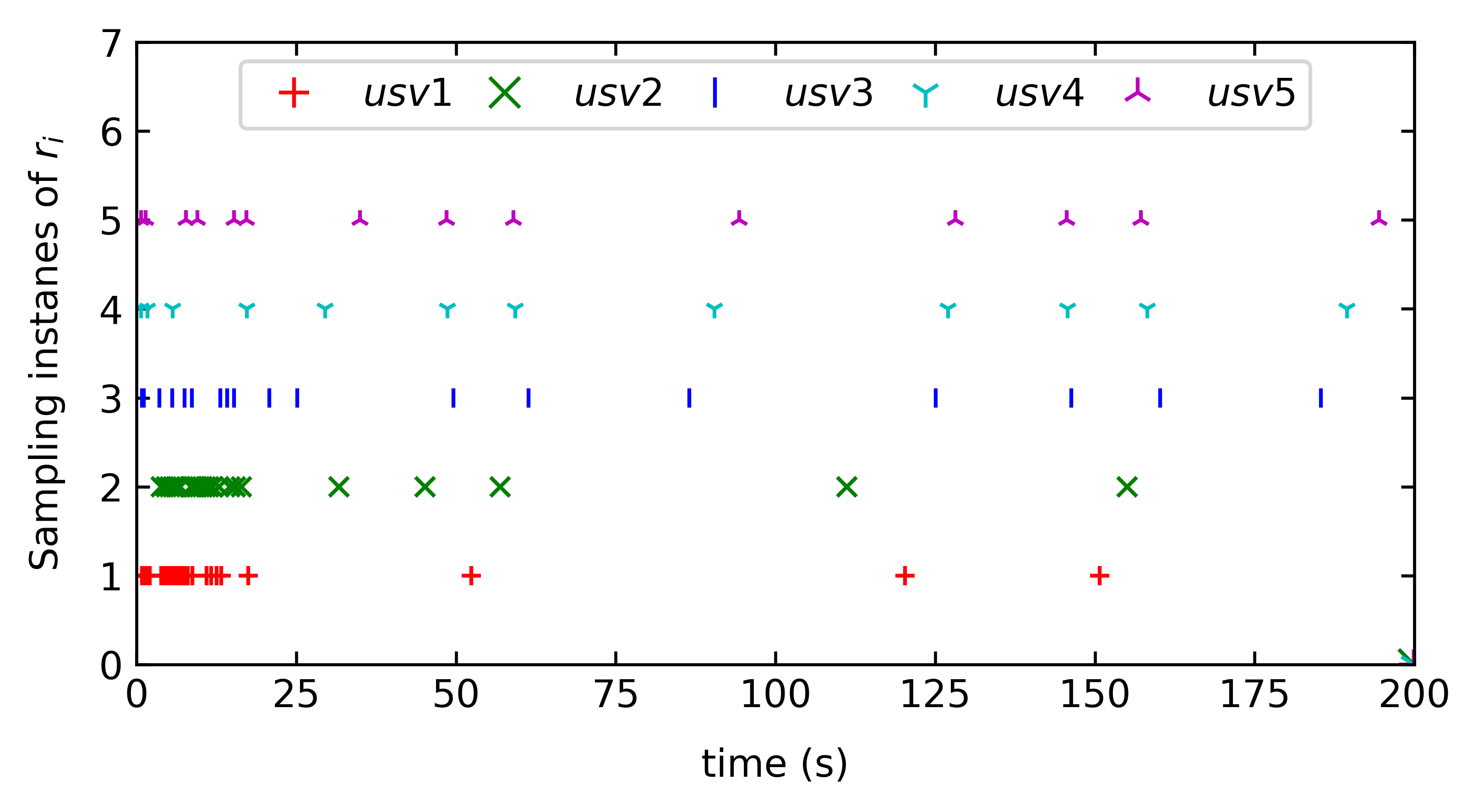


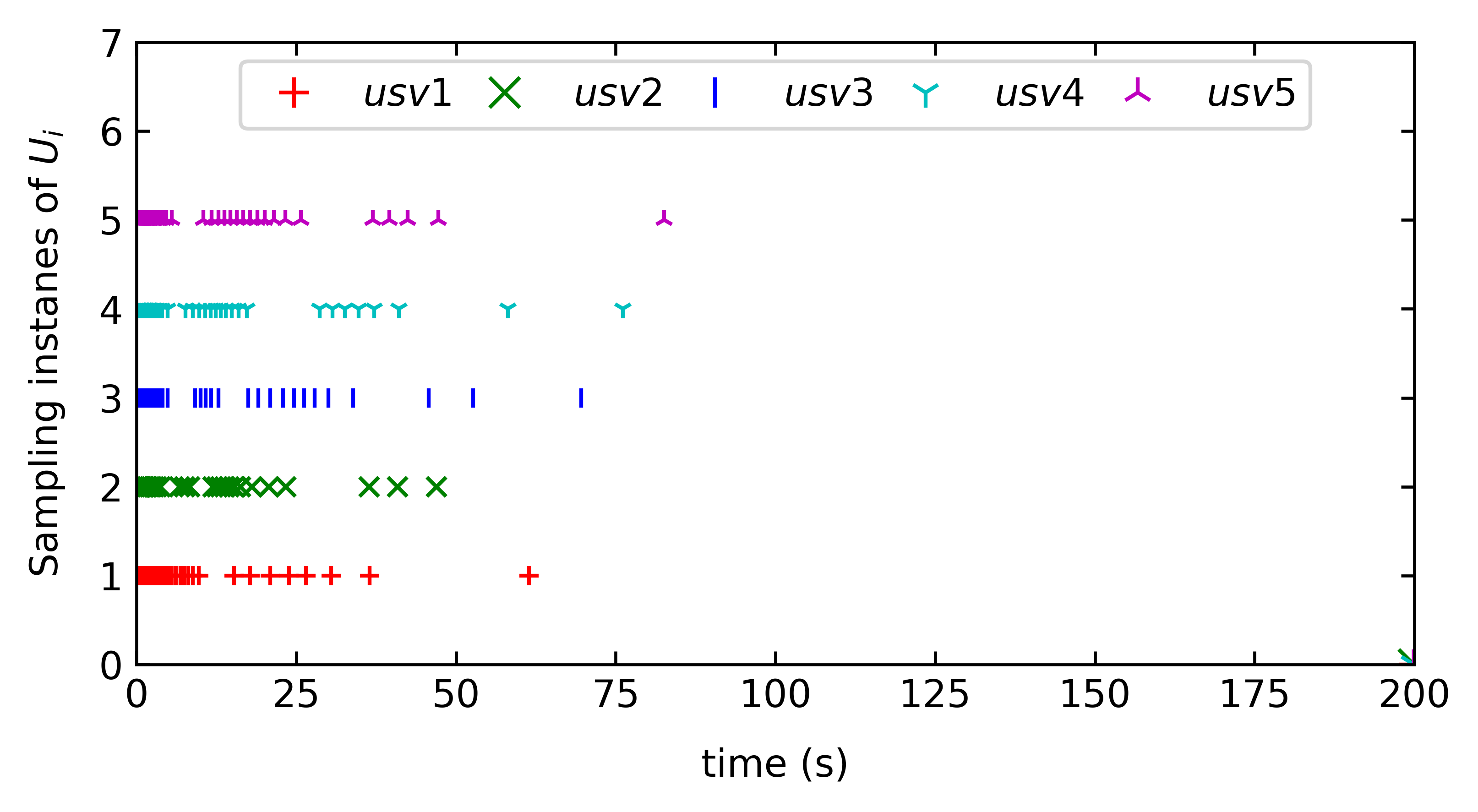


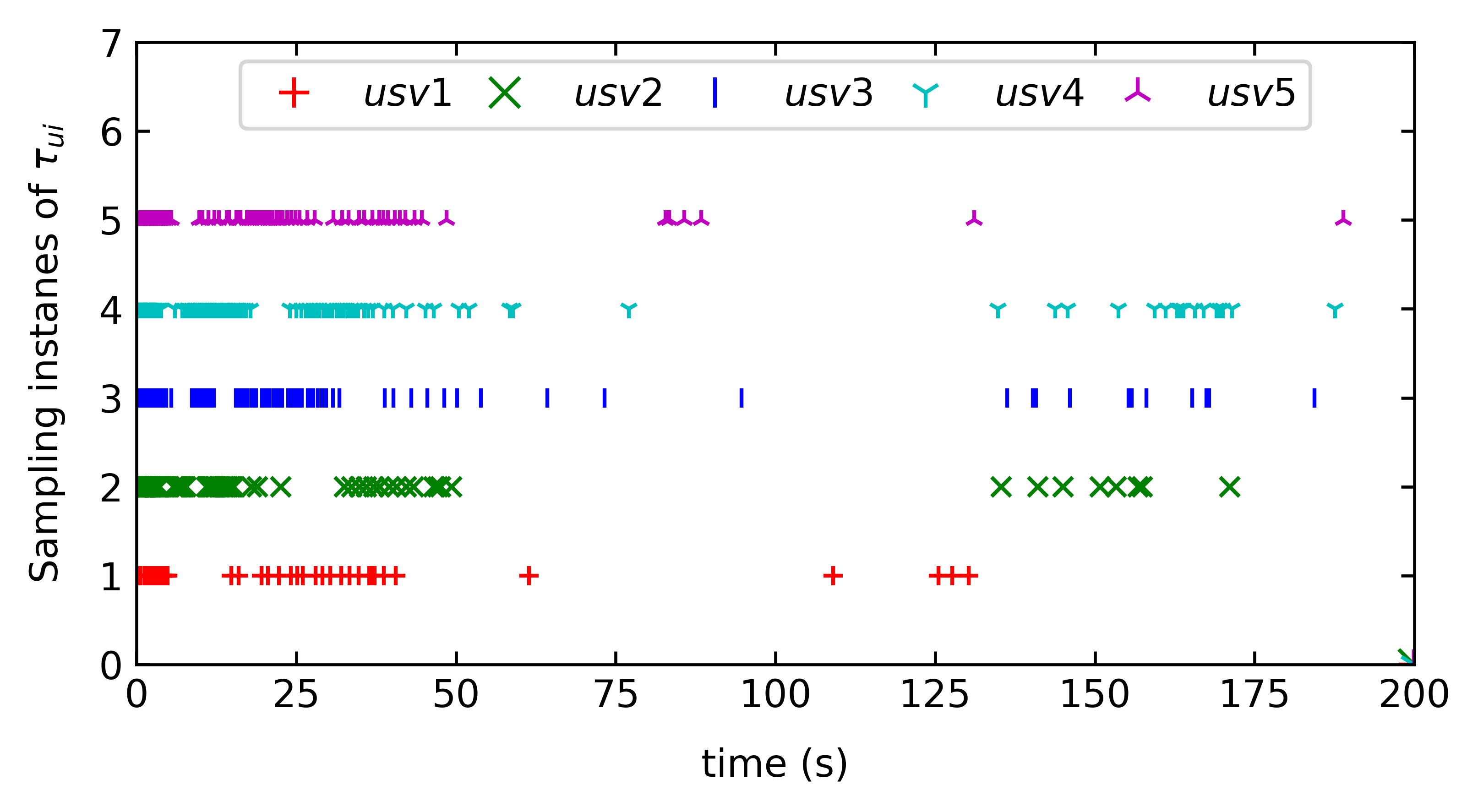


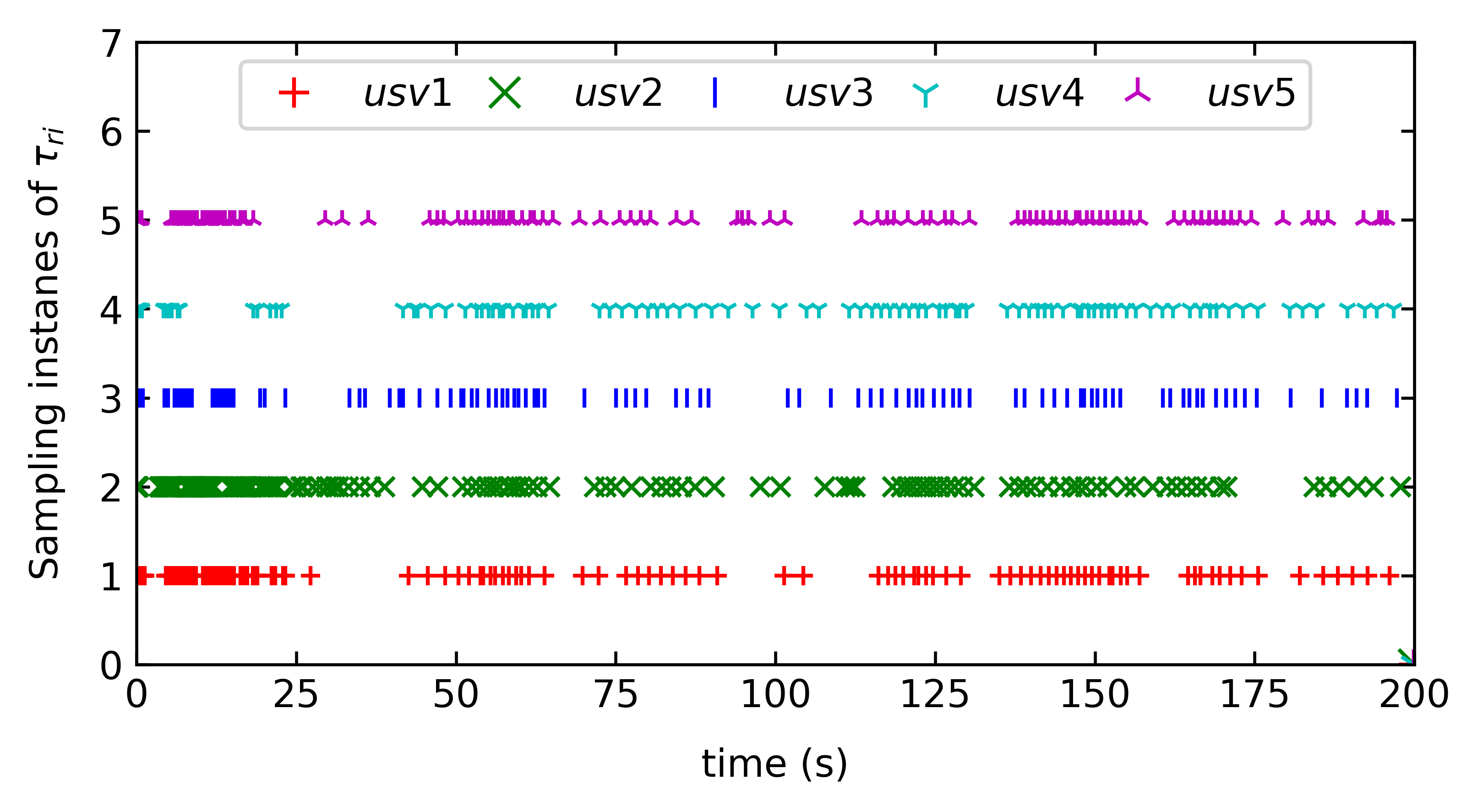


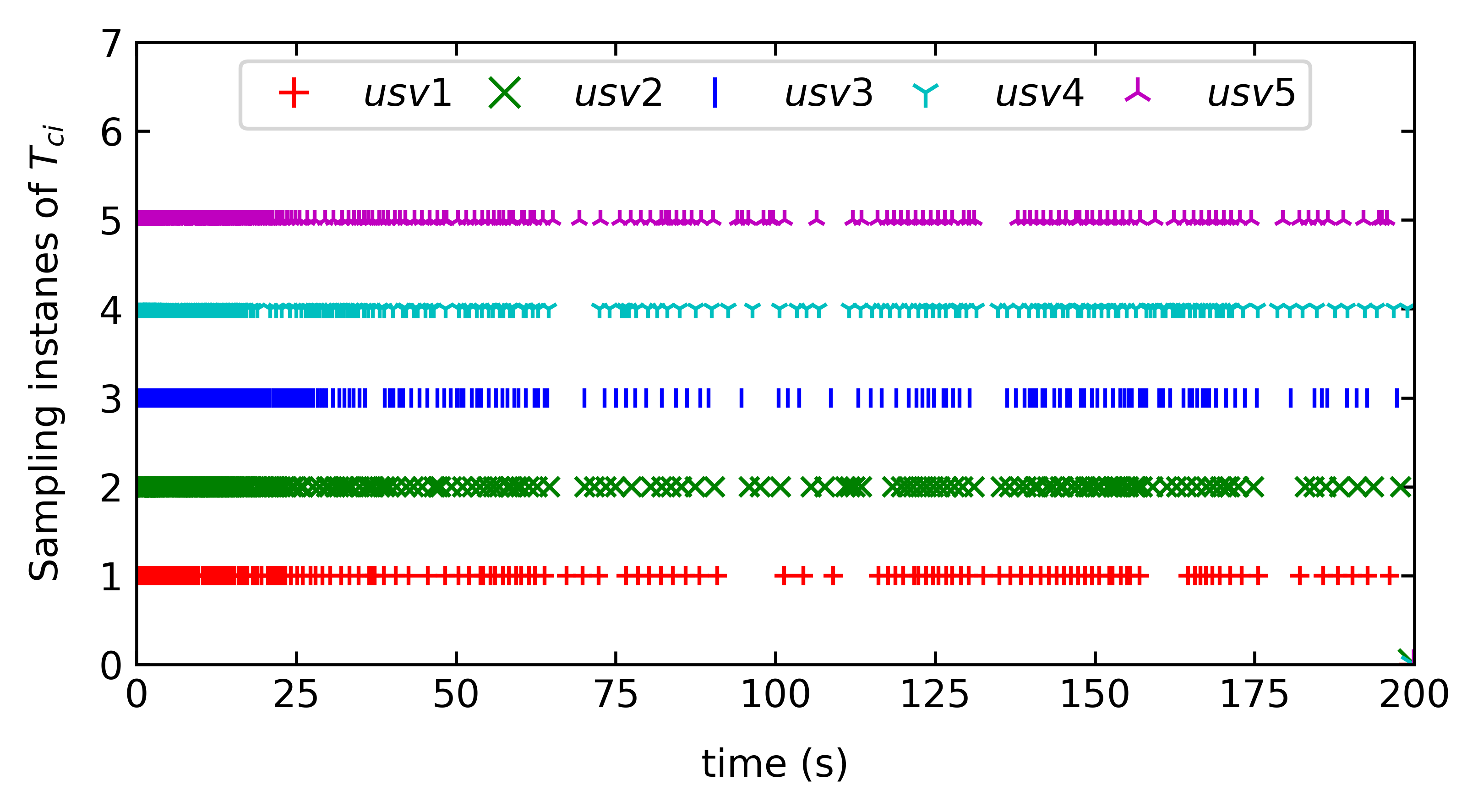












# Conclusion

# reference

2014年8月，美国海军进行了一次“蜂群”作战演示，依托雷达和红外传感器探测目标，以集群作战模式完成系列复杂动作，实施对“入侵目标”的包围和拦截，成功完成了护航作战任务。

2021年，美海军首次使用两艘MANTAS T-12无人艇与有人驾驶船只协同作战

2022年9月，大湾区近海海域，云洲智能研发的“17艘作业船+5艘无人船”配合，完成广东省自然资源厅部署的大湾区近海海域海底基础调查项目，将调查测绘效率提升了数倍。

2022年10 月，国家海上搜救综合演练在广东珠江口水域举行，模拟一艘客船与一艘油船在海上发生碰撞，导致客船起火、部分人员落水，油船船载油品泄漏、船舶失控，威胁深中通道施工水域在建大桥安全。面临两船相撞、多人落水、油品泄露、船舶失控的模拟“巨灾”情景，海上危机四伏，开展救援工作事故现场复杂。为快速搜寻落水人员，同时保障救援人员自身安全，云洲智能L90小型按要求高速自主航行，迅速抵达目标水域。凭借灵活、机动的特点，圆满完成搜救任务。

根据用于协调的导引信号的不同可将协调控制分为基于路径导引、基于轨迹导引、基于目标导引的协调控制，相较于后两种协调控制，基于路径导引的协调运动的控制器设计更为简单，产生的运动轨迹和执行器响应更为平滑，因此其在协调控制中应用较为广泛。 本文将对基于路径导引的欠驱动水面船协调控制问题开展研究。

其方法直观、计算复杂度低、跟踪效果好且易于工程实现，故其在路径跟踪领域应用最为广泛。视线制导（LOS）：关于视线制导的研究有很多，为克服传统视线制导的局限性，[4]中提出ILOS制导策略，ILOS在直线路径跟踪中效果很好，但其并不适用于曲线路径跟踪。针对参数化曲线路径的跟踪，[6]中提出了基于估计器的LOS制导（PLOS）方法，结合跟踪误差动态方程设计估计器，其可以很好的估计未知时不变小漂角，在[7]中基于PLOS提出了有限时间收敛的PLOS制导率（FPLOS），使其估计误差能够在有限时间内收敛，在[8]中进一步提出了固定时间收敛的PLOS制导率，和FPLOS相比，其收敛时间与初值无关，收敛速度快于PLOS。考虑输出误差约束及性能要求，在[10]和[11]中分别利用tan型李雅普诺夫函数和指定预设性能函数来约束路径跟踪误差，保证航行安全。

2014年8月，美国海军在弗吉尼亚州詹姆斯河上进行了一次“蜂群”作战演示，共有13艘无人艇参加，它们在收到直升机提供的威胁报警后，依托雷达和红外传感器探测目标，以集群作战模式完成了一系列复杂动作，实施了对“入侵目标”的包围和拦截，成功完成了护航作战任务。

2021年10月26日，美海军第59特遣队和巴林海军舰艇进行了“新地平线”(New Horizon)海上演习，期间美海军首次使用两艘MANTAS T-12无人艇与有人驾驶船只协同作战

2018年3月，56艘云洲无人艇组成的海上编队完成无人艇集群队形保持等多项测试科目

2022年9月，大湾区近海海域，“17艘作业船+5艘无人船”默契配合，不久前完成了广东省自然资源厅部署开展的今年首次大湾区近海海域海底基础调查项目。据悉，完成任务的无人船来自广东本土企业珠海云洲智能科技股份有限公司，首次参与这一任务，将调查测绘效率提升了数倍。